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## **AUTONOMOUS FORMATION FLYING - TANDEM-X, PRISMA AND BEYOND**

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### **ABSTRACT**

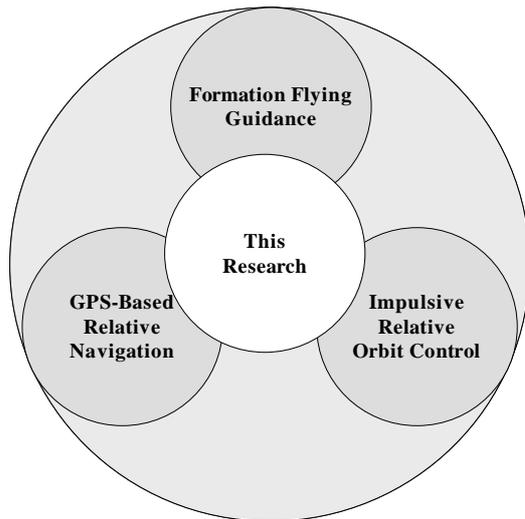
Formation flying is commonly identified as the collective usage of two or more cooperative spacecraft to exercise the function of a single monolithic virtual instrument. The distribution of tasks and payloads among fleets of coordinated smaller satellites offers the possibility to overcome the classical limitations of traditional single-satellite systems. The science return is enhanced through observations made with larger, configurable baselines and an improved degree of redundancy can be achieved in the event of failures. Different classes of formation flying missions are currently under discussion within the European engineering and science community: technology demonstration missions (e.g. PRISMA, PROBA-3), synthetic aperture interferometers and gravimeters for Earth observation (e.g. TanDEM-X, postGOCE), dual spacecraft telescopes which aim at the detailed spectral investigation of astronomical sources (e.g., XEUS, SIMBOL-X), multi-spacecraft interferometers in the infrared and visible wavelength regions as a key to new astrophysics discoveries and to the direct search for terrestrial exoplanets (e.g., DARWIN, PEGASE). These missions are characterized by different levels of complexity, mainly dictated by the payload metrology and actuation needs, and require a high level of on-board autonomy to satisfy the continuously increasing demand of relative navigation and control accuracy.

In order to respond to this demand the DLR's German Space Operations Center (GSOC) is pursuing a dedicated autonomous formation flying research and development roadmap since 1998. The research work has largely been motivated by the conviction that only the development, deployment and on-orbit validation of innovative Guidance, Navigation and Control (GNC) techniques can bring formation flying to the forefront and enable the definitive transfer of this revolutionary technology to space. As a result the GSOC's contributions to TanDEM-X and PRISMA (both launches expected in 2009) will demonstrate, for the first time in Europe, autonomous fuel-efficient formation keeping and reconfiguration on a routine basis, with minimum collision risk. After a comprehensive introduction on the state-of-the-art of the formation flying technology in Low Earth Orbit (LEO), the paper addresses the design, implementation and testing of the DLR/GSOC's GNC subsystems for TanDEM-X and PRISMA demonstration missions. An overview of the developed subsystems is provided, highlighting communalities and differences of the two parallel developments. Furthermore key results from the validation of the guidance strategy, of the real-time GPS-based navigation and of the impulsive relative orbit control functions are presented.

A technological gap clearly exists between the remote sensing LEO formations, yet to be demonstrated, and the planned outer space distributed telescopes in high elliptical orbits or in the vicinity of the Lagrange points. It is not only given by the envisaged three-order-of-magnitude improvement of the required metrology and actuation needs, but is also driven by the necessity of implementing navigation systems at altitudes above the GNSS constellations. The final part of the paper is thus devoted to the identification of the major discrepancies between present and next generation formation flying. An attempt is made to define the way forward and offer an outlook beyond the first European technology demonstration missions.

## 1. STATE OF THE ART

In general the research in the area of formation flying is characterized by a high level of multidisciplinary. As illustrated in Fig. 1, many authors have worked independently on different disciplines like guidance, navigation and control of multiple satellites but little effort has been put into the design of a complete functional subsystem to be embedded in a spacecraft platform.



**Fig. 1: The research at DLR/GSOC brings together and builds upon the work done by many others in the areas of formation flying guidance [Clohessy and Wiltshire, 1960; Eckstein et al., 1989; Sabol et al., 2001; Schaub, 2004; D'Amico and Montenbruck, 2006], GPS-based relative navigation [Kawano et al., 1999; Ebinuma, 2001; Busse, 2003; Leung, 2003; D'Amico et al., 2006a] and impulsive relative orbit control [Micheau, 1995; Schaub and Alfriend, 2001; Vaddi et al., 2005; D'Amico et al., 2006b].**

This section is devoted to the presentation of the body of knowledge and experience gained in the past years in three distinctive areas, namely formation flying guidance, GPS-based relative navigation and impulsive relative orbit control. The selection of specific methods like GPS and impulsive control reflects the real applications under examination in this work and hides some relevant design choices that are strictly related to the kind of problem we want to solve.

GPS is the primary source of relative navigation in LEO. GPS provides highly accurate timing information for on-board time synchronization, enables simultaneous measurements from the

spacecraft within the formation and offers the required level of accuracy in the context of carrier-phase differential GNSS (CDGNSS) techniques [Leung and Montenbruck, 2005], [Ebinuma, 2001], [Hartrampf et al., 2002], [Kroes et al., 2005].

Impulsive orbit control is nowadays the only feasible option considering the limitations imposed by the current propulsion technologies and by the sensitivity of scientific instruments to external accelerations. In general, thruster activities have to be minimized to maximize the available time for data collection, thus orbit maintenance maneuvers are realized in short time intervals so to maximize the thrust-free portion of the spacecraft trajectories [Scharf et al., 2002].

### 1.1 Formation Flying Guidance

Various parameterization methods have been used in the past to describe the relative motion between co-orbiting spacecraft in a formation. The aforementioned C-W equations utilizes a Hill-like [Hill, 1878] rotating Cartesian coordinate frame with origin on the chief satellite to express the relative position and velocity vectors of a deputy satellite. Hence the other name of Hill's equations often used together with C-W. Curvilinear coordinates in the same Hill frame have also been adopted to derive more accurate expressions of the relative motion. The linear formulation provided by the C-W equations assumes small deviations from a circular reference orbit about a spherical Earth. Considerable effort has been put into the generalization of these equations to include disturbance forces [Sabol et al., 2001], [Schweighart and Sedwick, 2004], and eccentric reference orbits [Inalhan et al., 2002], [Gim and Alfriend, 2001].

The six Cartesian initial conditions are the invariant parameters of the relative motion described by the C-W equations. As originally suggested by Hill in his development of the lunar theory [Hill, 1878], an alternate set of six invariant parameters can be used to conveniently express the motion relative to the reference orbit (i.e., relative to the chief spacecraft). For a Keplerian motion, as assumed by the C-W equations, the chief and deputy satellite trajectories are conics conveniently defined by a Lagrange set of orbital elements:  $a$ ,  $e$ ,  $i$ ,  $\omega$ ,  $\Omega$ , and  $M_0$  which

denote semi-major axis, eccentricity, inclination, argument of perigee, right ascension of the ascending node and the mean anomaly at the initial time  $t_0$ . The existence of invariant parameters of the absolute motion (the Lagrange orbital elements) and the well known non-linear one-to-one mapping between these parameters and the Cartesian position and velocity at the same instant of time pushed several authors to search combinations of orbital elements as constants of the relative motion [Eckstein et al., 1989], [Kasdin and Gurfil, 2003], [Schaub, 2004]. In the following these invariant parameters are referred to as relative orbital elements to distinguish them from the classical Keplerian (or absolute) orbital elements.

The relative orbital elements found their first definitions and applications in the years 1980–1990 when the full potential of the Geostationary Orbit (GEO) as provider of facilities for communications and Earth observation was recognized on a global scale. The growing trend in telecommunications and thus the huge demand for orbital positions in GEO induced the development of the concept of satellite collocation. Several satellites had to be positioned in so-called GEO windows usually reserved for one satellite only. The interference between satellites due to the high probability of close approaches and collision risks had to be mitigated through coordinated orbit control strategies in order to separate the satellites physically in space whilst still maintaining them within the nominal window.

Various approaches to the separation of colocated satellites have been developed, such as introducing a time interval between the application of station keeping maneuvers for each satellite or operating each satellite in a slightly non-GEO orbit by selecting appropriate differences in the orbital elements (e.g., using a slightly different eccentricity or inclination), or a combination of the two [Eckstein et al., 1989], [Härting et al., 1988]. More recently the clear advantage of the parameterization of the relative motion in terms of relative orbital elements has been recognized by various authors. This approach provides direct insight into the shape, size and location of the formation geometry and allows the straightforward adoption of variational equations such as Lagrange's planetary equations or Gauss's variational equations to

study the effects of orbital perturbations on the relative motion. Kasdin and Gurfil have tried to unify the merits of the C-W and orbital elements-based approaches by developing a Hamiltonian methodology that models the relative motion dynamics using canonical coordinates, termed "epicyclic" elements [Kasdin and Gurfil, 2003]. A lower level of abstraction is finally presented by Schaub who has extensively examined the relative orbit geometry through classical orbital element differences [Schaub, 2004]. Here direct linearized relationships between classical orbital elements differences and the resulting relative orbit geometry are presented for both circular and eccentric (chief) orbits with the incorporation of the gravitational perturbation resulting from the Earth's flattening.

## 1.2 GPS-Based Relative Navigation

A fundamental need of the spacecraft autonomous formation flying is the determination of the relative motion (i.e., position and velocity) between individual satellites in near real-time. For formation flying in LEO, differential GPS (DGPS) represents an ideal sensor which can be used to directly measure the relative positions and velocities to a high level of accuracy with low costs. In particular raw measurements of carrier phase and pseudo-range from two or more user spacecraft made to common GPS satellites in the constellation can be subtracted from each other to reduce systematic errors. Compared to typical raw measurements, differenced GPS observation data have a high level of common error cancellation and, as a consequence, are less sensitive to GPS satellite clock offsets, GPS broadcast ephemeris errors, ionospheric refraction, and biases due to hardware delays. Many authors have recognized these advantages and worked on GPS-based relative navigation of space vehicles since the late 1990s. Relative navigation accuracies at the meter level have first been demonstrated in earlier missions like the Automated Transfer Vehicle (ATV) Rendezvous Predevelopment Program (ARP) [Carpenter, 2001], and the ETS-VII mission [Kawano et al., 1999] by making use of differenced pseudo-range measurements only. Recent hardware-in-the-loop simulations using GPS signal simulators have performed much better, mainly because more sophisticated carrier phase differential

techniques have been used. Studies performed by Ebinuma [2001], Busse [2003], Hartrampf et al. [2002], Leung and Montenbruck [2005] have all demonstrated real-time relative navigation at the (sub-)centimeter level using single frequency GPS receivers. While achieving these results, each of the aforementioned authors has addressed different facets of the navigation problem. Ebinuma has demonstrated precise closed-loop rendezvous of two spacecraft and achieved a relative position accuracy of 5 cm (3D, rms) for baselines up to 10 km [Ebinuma, 2001]. His navigation filter processes double-difference carrier phase data and the relative state is computed from the difference between the two absolute state estimates. No integer ambiguity fixing to integer values is performed. A high performance desktop computer is used for real-time simulations. Busse has also achieved similar accuracies through a filter which directly estimates the relative states from single-difference carrier phase measurements and a known local absolute state [Busse, 2003]. The prototype code is tested through off-line analysis of raw GPS measurements recorded in a signal simulator test-bed. Hartrampf has demonstrated relative navigation with an accuracy of about 1 cm (3D, rms) in an ionosphere-free simulation scenario for a 1-km baseline [Hartrampf et al., 2002]. The relative navigation filter makes use of double-difference carrier phase data types in a purely kinematic manner by fixing the integer ambiguities. Real-time simulations are implemented on a standard desktop computer. The best results to date have been achieved by Leung and Montenbruck who demonstrated GPS-based real-time relative navigation accuracies at the sub-centimeter level for formations with 1–10 km baselines [Leung, 2003], [Leung and Montenbruck, 2005]. At the basis of this improved performance is the adoption of a more rigorous relative motion model, the resolution of double-difference integer ambiguities and the usage of a GPS receiver optimized for low-noise carrier phase tracking under space dynamics. Furthermore the authors used a convenient linear combination of pseudo-range and carrier phase termed GRoup And Phase Ionosphere Correction (GRAPHIC) for absolute navigation [Yunck, 1993]. This ionosphere-free GRAPHIC measurement is characterized by a lower noise and removes an otherwise significant source of error for single

frequency GPS users. In contrast to previous studies, Leung and Montenbruck have developed a real-time navigation system embedded in a realistic flight computer with its inherent limitations in memory and computing performance. The navigation accuracies have been evaluated in hardware-in-the-loop simulations comprising GPS signal simulators, GPS receivers, the navigation computer and radio modems.

### 1.3 Impulsive Relative Orbit Control

The control of satellite formations is efficiently performed by the activation of on-board thrusters. Typically, impulsive control, applied at proper locations, is preferred to thrust application for an extended period of time. This approach is not only justified because of the current limitations in propulsion technologies and because of the typical payload requirements [Gill and Runge, 2004]. Indeed impulsive-feedback-control laws can be designed analytically, give the possibility to exploit the natural orbital dynamics to its full extent and can be easily adopted for a ground-in-the-loop scheme or for an autonomous implementation. The Gauss' variational equations of motion offer the ideal mathematical framework for designing impulsive control laws [Battin, 1987], [Micheau, 1995]. These equations have been extensively used in the last decades for absolute orbit keeping of single spacecraft, but only recently are being exploited for formation flying control in LEO [Schaub and Alfriend, 2001], [Vaddi et al., 2005]. The reason for such an evident delay is that the Gauss' variational equations provide relationships between the control acceleration and the time derivative of the orbital elements which were normally used to parameterize the motion of an individual satellite but not the relative motion of a formation. After the advent of the first characterizations of the relative motion in terms of relative orbital elements (cf. Sec. 1.1), many authors realized how natural and convenient was the adoption of the Gauss' variational equations for formation flying applications. Mainly Alfriend [Alfriend et al., 2003], Schaub [Schaub and Alfriend, 2001], Vadali [Vadali et al., 1999] and Vaddi [Vaddi et al., 2005] have addressed the problem of impulsive relative orbit control for formation establishment and reconfiguration, in presence of  $J_2$  Earth's oblateness effects and for  $J_2$

invariant formations. The works mentioned clearly demonstrate the high potential of impulsive orbit control for formation-flying applications but do not provide a realistic validation of the fundamental algorithms in terms of accuracy and robustness. The authors make use of pure software simulations in an ideal perturbation environment (e.g., neglecting differential drag). Issues like sensors, actuators and on-board implementation of the algorithms in real-time are ignored. A few studies exist on formation keeping using impulsive orbit control which make use of Cartesian coordinates feedback laws instead of relative orbital elements [Middour, 1991], [Wiesel, 2003]. The control laws developed by Middour acquire and maintain the desired along-track separation through impulsive maneuvers in the along-track direction only [Middour, 1991]. Wiesel addressed the theory of optimal impulse control of relative satellite motion and solved numerically the resulting optimization problem [Wiesel, 2003]. In general the element feedback based control is typically applied in a pulse like manner, the Cartesian coordinate feedback normally demands a continuous thrust. The reason for this is that the orbital elements errors are very slowly varying quantities if compared with the position and velocity vector errors. This is also the reason why it is more difficult to bring the thrust magnitudes within practical constraints when dealing with Cartesian position and velocity.

## 2. CONTRIBUTIONS OF THIS RESEARCH

The research at DLR/GSOC aims at the first realistic demonstration of a complete GNC system for formation flying spacecraft in LEO. Numerous technical contributions have been made in the areas of formation flying guidance [D'Amico and Montenbruck, 2006], [D'Amico et al., 2005], GPS based relative navigation [D'Amico et al., 2006a], and impulsive relative orbit control [D'Amico et al., 2006b], but the primary contribution does not lie in one or more of these disciplines. The innovation and originality of the work stems from the design and implementation of a comprehensive formation flying system through the successful integration of various techniques [Gill et al., 2007].

The research activities at DLR/GSOC have led to the full development, testing and validation of the GNC flight code to be embedded in the on-

board computer of the active spacecraft of the PRISMA technology demonstration [D'Amico et al., 2008]. Furthermore novel guidance and control algorithms are going to be demonstrated for the first time in the TerraSAR-X/TanDEM-X formation flying mission [Ardaens et al., 2007].

Overall the DLR/GSOC roadmap in the frame of autonomous formation flying focuses on realistic application cases closely related to upcoming formation flying missions. The intention is to realize a practical and reliable way to formation flying: a technology that is discussed and studied since decades but is still confined in research laboratories. Hardware-in-the-loop real-time simulations, including a representative flight computer and the GPS hardware architecture, show that simple techniques, which exploit the natural orbit motion to full extent, can meet the demanding requirements of long-term close formation flying. For completeness the following sections give a detailed summary of the relevant contributions of this research to the body of knowledge.

### 2.1 Formation Flying Guidance and Control

Although using the Hill frame coordinates is a common method to describe the satellites relative motion, they have the distinct disadvantage that for a general orbit the differential equations of motions must be solved to obtain the precise instantaneous geometry of the formation. Because of this fact, a description in terms of relative orbital elements has been preferred to the canonical Cartesian parameterization. In contrast to the fast varying position and velocity variables, the use of orbital element differences simplifies the formation-flying description and the satellite relative position computation. Various sets of relative orbital elements have been proposed in the past decades in the frame of formation-flying dynamics and control, but actually the most intuitive, straightforward representation in terms of relative eccentricity and inclination vectors has never been investigated for formation-flying design in LEO. This research generalizes the method of eccentricity/inclination vector separation, first developed for the safe collocation of geostationary satellites, and extends its application to proximity operations of formation-flying spacecraft. The spontaneous geometrical representation offers a direct

correlation between the relevant characteristics of the bounded relative motion in near circular orbit and the magnitude/phase of the relative eccentricity/inclination vectors. This aspect extremely simplifies the design of safe, passively stable formation-flying configurations. In particular minimum collision risk conditions can be guaranteed by imposing the (anti-) parallelism of the eccentricity and inclination vectors of the respective satellites, while  $J_2$ -stable relative orbits are obtained by setting a specific nominal phase for the configuration. The new approach is shown to be suitable either for the realization of SAR interferometers with baselines below 1 km or the application in longitude swap operations with along-track separations above 200 km [D'Amico and Montenbruck, 2006], [Montenbruck et al., 2006].

In the first case an active relative orbit control strategy is necessary, in order to compensate for the main disturbance forces represented by Earth's oblateness perturbations and differential aerodynamic drag. The proposed strategy is based on the eccentricity/inclination vectors control and makes use of pairs of pulses separated by half a revolution. The method is very simple and can be used for a ground-in-the-loop control system as well as for an autonomous on-board implementation. The required velocity budget for formation-keeping can be expressed in terms of relative orbital elements and is directly proportional to the relative eccentricity and inclination offsets. Furthermore the proposed two-impulse analytical solution is adopted to reconfigure the formation in a safe and fuel-efficient way.

## 2.2 GPS-Based Relative Navigation

The results obtained so far by various authors demonstrate that CDGPS is an invaluable source of relative navigation in LEO. The use of space-borne GPS receivers hardware and true GPS signals in their hardware simulations marks a major progress on the way to acquire flight experiments. Nevertheless some limitations characterize the previous studies and, as a consequence, have been addressed at DLR/GSOC. First of all, the presented prototype navigation systems do not incorporate maneuvers, which will be crucial for use in orbital control and formation-keeping. Secondly the handling of the spacecraft attitude and the robustness of the filter to non-ideal non-zenith orientations of the GPS antennas are

neglected. Last but not least contingency scenarios or delicate formation-flying operations phases like the Launch and Early Operations Phase (LEOP) or the safe separation of the spacecraft from a common combined configuration are typically not addressed.

The weakness of previously designed filters makes the strength of this development. One of the main challenges of a real formation flying is the realization of an on-board navigation system for all mission phases which is robust and accurate even for various spacecraft orientations and frequent thruster firing for orbit control. In contrast to earlier approaches that separate the GPS-based navigation into the independent reconstruction of absolute and relative states, a single reduced-dynamic Kalman filter for the absolute states of both spacecraft has been adopted. Two different types of measurements are processed by the filter: undifferenced GRAPHIC measurements of the individual spacecraft as well as single-difference carrier-phase measurements. GRAPHIC denotes an ionosphere-free linear combination of pseudo-range and carrier phase data. It enables an absolute orbit determination of each individual spacecraft with a representative accuracy of about 1–2 m, whenever a sufficient number of GPS satellites is tracked. The single-difference carrier phase measurements in contrast can only be formed for commonly observed GPS satellites but exhibit a much lower noise level of ca. 1–2 mm and thus provide the relative orbit with much higher accuracy. Both data types are subject to ambiguities related to the nature of carrier phase measurements. Channel specific ambiguities must therefore be estimated as part of the navigation filter. However, no effort is made to fix double-difference ambiguities to integer values. In view of residual modeling uncertainties (caused, for example, by the limited knowledge of the spacecraft attitude and antenna position) the benefits of ambiguity fixing cannot be materialized in a practical real-time navigation filter. Overall, a total of 49 parameters are estimated in the navigation filter. These comprise the position/velocity vector, empirical accelerations, drag coefficient and clock offset as well as a total of 12 GRAPHIC bias parameters for each of the two spacecraft. In addition, the filter state is augmented by a 3-parameter delta-v vector to enable the estimation of impulsive velocity increments after maneuvers. The inherent

robustness of the symmetric filter design originates from the fact that common GPS satellites visibility is not a prerequisite to reconstruct the relative state. Even in the case of spacecraft with completely different attitude, the relative state can be determined by simply differencing absolute estimates exclusively based on GRAPHIC data types [Gill et al., 2007]. The unified filter design simplifies the initialization and the maneuver handling procedures, and, consequently, improves the flexibility of the navigation system and its reliability during formation flying experiments. A Runge-Kutta fourth order integrator with Richardson extrapolation and Hermite interpolation allows the provision of continuous position and velocity data at a 1 Hz rate and gives the possibility to efficiently cover the GPS data gaps caused by the tumbling of the chief spacecraft during the early separation phase. Moreover the developed GPS system is able to incorporate orbit control maneuvers in the navigation process. This feature enables not only the absorption of the velocity variations imparted to the two spacecraft by the separation mechanism but also their estimation via the Kalman filter state.

### 3. DEMONSTRATION MISSIONS

A dedicated research and development program on autonomous spacecraft navigation and formation flying was initiated at the German Space Operations Center (GSOC) of DLR (German Aerospace Center) in 1998. Numerous contributions in the area of spaceborne GPS receiver technology, precision relative navigation and autonomous orbit control of satellite formations as a prerequisite for spacecraft autonomy have been made.

Practical experience in the operations of a two-satellite formation has been gained by the GRACE mission [Kirschner, 2003]. Next, GSOC is supporting the Swedish Space Cooperation (SSC) in the implementation of the PRISMA formation flying demonstration mission, where a fully autonomous, robust and accurate formation flying of spacecraft will be conducted by several experiments [Persson et al., 2006]. Finally, the TerraSARX/TanDEM-X radar satellites will be Europe's first space mission equipped and operated routinely with an autonomous formation flying system [Ardaens et al., 2007]. Most of the conducted work is motivated by and find practical application in

the aforementioned projects. Therefore an overview of the PRISMA and TerraSAR-X/TanDEM-X missions is provided in the following.

#### 3.1 TanDEM-X

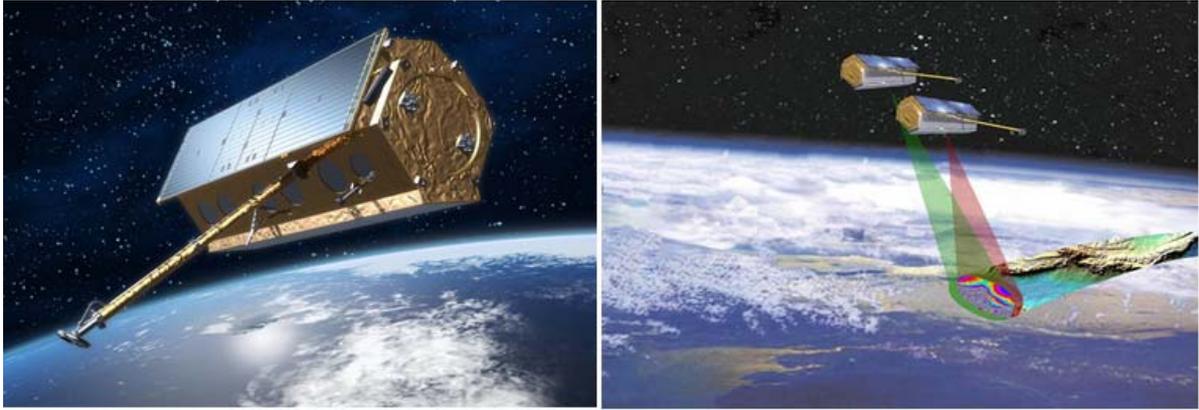
##### 3.1.1 TanDEM-X Mission

TerraSAR-X (TSX) is an advanced SAR-satellite system for scientific and commercial applications, which is realized in a Public-Private Partnership (PPP) between DLR and EADS Astrium GmbH. The satellite has a size of 5 m x 2.4 m, a mass of 1341 kg and carries a high-resolution SAR operating in the X-band (9.65 GHz). A Russian DNEPR-1 rocket launched from Baikonur, Kazakhstan, has injected TerraSAR-X into a 514 km sun synchronous dusk-dawn orbit with 97° inclination and an 11 day repeat period. TerraSAR-X is planned to be operated for a period of 5 years and will therefore provide SAR-data on a long-term, operational basis. DLR/GSOC will provide the Mission Operations Segment (MOS) using ground stations at Weilheim and Neustrelitz.

As a complement to TSX, the TanDEM-X (TDX) mission is under development in the frame of new Earth observation missions within the German national space program (cf. Fig. 2). It involves a second spacecraft, which is almost identical to TerraSAR-X and shall likewise be operated for 5 years starting from mid 2009. The two spacecraft will fly in a precisely controlled formation to form a radar interferometer with typical baselines of 1 km. This allows a much higher resolution than achievable in the X-SAR/SRTM Shuttle Topography mission and thus the generation of digital elevation models (DEMs) with unrivaled accuracy.

##### 3.1.2 TanDEM-X Autonomous Formation Flying (TAFF)

TanDEM-X will be equipped with an Autonomous Formation Flying (TAFF) system developed by DLR/GSOC. This offers a unique chance to both enhance and intensify the knowledge and experience in the area of formation flying. Furthermore, the implementation of autonomous formation flying functionalities on the TDX spacecraft is considered to be a key driver for a more



**Fig. 2: Artist's impression of the TerraSAR-X spacecraft (left) and the TerraSAR-X/TanDEM-X formation (right). Courtesy of EADS Astrium GmbH.**

efficient use of the available on-board resources.

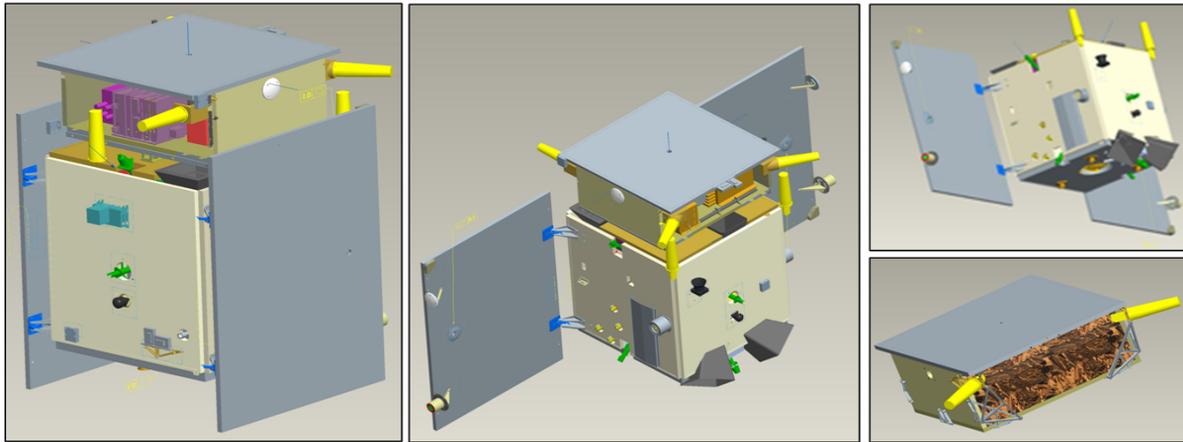
The objective of TAFF is to enable a simple and robust formation control in order to ease on-ground operations. To achieve this goal dedicated functions for formation guidance, navigation and control (GNC) will be implemented on-board TanDEM-X. Navigation will employ GPS data from the Mosaic GNSS receivers on-board TanDEM-X and TerraSAR-X. TSX GPS data will be provided through a dedicated S-Band ISL. Instead of raw code and phase measurements, TAFF will make use of the dynamically filtered GPS position fixes. These pseudo-measurements are then processed in a Kalman filter employing a dynamical model of relative motion which consists, in its simplest form, of the analytical Hill-Clohessy-Wiltshire (HCW) equations. Since guidance and control are based on the parameterization of the relative motion in terms of relative orbital elements, the same parameterization is applied in the navigation function to achieve a consistent formulation of the overall GNC functions in TAFF. As a result, the relative orbital elements are estimated by the TAFF navigation function and are output to the guidance and control functions. Guidance and control require, in addition to the relative trajectory, absolute trajectory information as well. This is realized within TAFF by the direct conversion of TDX navigation solutions to orbital elements without any filtering. Since the navigation solution output from the MosaicGNSS receiver has already been filtered internally, no post-filtering is required. Furthermore, the filtering within the MosaicGNSS receiver assures a continuous output of solutions even in the presence of GPS

data gaps which avoids the need for an absolute orbit filter in TAFF and significantly simplifies its design. The robustness of the formation control will be achieved by guidance and control functions which are based on the separation of relative eccentricity and inclination vectors. This allows a robust formation configuration with minimum collision risk. TerraSAR-X/TanDEM-X will furthermore be the first operational mission requiring a post-facto baseline reconstruction with an accuracy of 1 mm. The feasibility of achieving this goal using GPS dual-frequency measurements of the IGOR GPS receiver has earlier been demonstrated in the GRACE mission [Kroes et al., 2005]. The respective algorithms will further be refined and adapted to benefit from the small separation of the two spacecraft, which would even allow a single-frequency integer ambiguity resolution. Furthermore, the impact of phase pattern variations will be addressed through dedicated calibration campaigns of the antenna system.

## 3.2 PRISMA

### 3.2.1 PRISMA Mission

PRISMA is a Swedish led mission with DLR contributions in the area of GPS based navigation and autonomous formation flying. The mission objectives of PRISMA may be divided into the validation of sensor and actuator technologies related to formation flying and the demonstration of experiments for formation flying and rendezvous. Key sensor and actuator components [Persson et al., 2006] comprise a GPS receiver system, two vision based sensors (VBS), two formation flying radio



**Fig. 3: Artist's impression of the PRISMA clamped configuration after launch (left), with the *Main* solar panels deployed (center), and the individual *Main* (right-top) and *Target* (right-bottom) spacecraft when separated. Courtesy of Swedish Space Corporation (SSC).**

frequency sensors (FFRF), and a hydrazine mono-propellant thruster system (THR). These will support and enable the demonstration of autonomous spacecraft formation flying, homing, and rendezvous scenarios, as well as close-range proximity operations.

The mission schedule foresees a launch of the two spacecraft in June 2009. The spacecraft are named *Main* and *Target* and will be injected by a DNEPR-1 launcher into a sun-synchronous orbit at 700-km altitude and 98.2° inclination. A dusk-dawn orbit with a 18 h nominal local time at the ascending node (LTAN) is targeted. Maximum eclipse times of 23 minutes may occur for injections within  $\pm 1$  h off the nominal LTAN, depending on the sun's declination.

Following a separation from the launcher, the two spacecraft will stay in a clamped configuration for initial system checkout and preliminary verification (cf. Fig. 3). Once the spacecraft are separated from each other, various experiment sets for formation flying and in-orbit servicing will be conducted within a minimum targeted mission lifetime of eight months. Spacecraft operations will be performed remotely from Solna, near Stockholm, making use of the European Space and Sounding Rocket Range (Esrange) ground station in northern Sweden. The S-band ground-space link to *Main* supports commanding with a bit rate of 4 kbps and telemetry with up to 1 Mbps. In contrast, communication with the *Target* spacecraft is only provided through *Main* acting as a relay and making use of a *Main-Target* intersatellite

link (ISL) in the ultrahigh-frequency (UHF) band with a data rate of 19.2 kbps.

The *Main* spacecraft has a wet mass of 150 kg and a size of 80 × 83 × 130 cm in launch configuration. In contrast to the highly maneuverable *Main* spacecraft, *Target* is a passive and much simpler spacecraft, with a mass of 40 kg at a size of 80 × 80 × 31 cm (cf. Fig. 3). Electrical power for the operation of the *Main* spacecraft bus and payload is provided by two deployable solar panels delivering a maximum of 300 W, whereas *Target* relies on one body-mounted solar panel providing a maximum of 90 W. The *Main* spacecraft implements a three-axis, reaction-wheel based attitude control and three-axis delta-v capability. To this end, the *Main* GNC sensors comprise two three-axis magnetometers (MM), one pyramid sun acquisition sensors and five sun-presence sensors (SS), five single-axis angular-rate sensors (GYR), five single-axis accelerometers (ACC), two star-tracker camera (SCA) heads for inertial pointing, two GPS receivers, two vision-based sensors (VBS) and two formation flying radio frequency sensors (FFRF). As actuators, three magnetic torque rods (MT), four reaction wheels (RW), and six thrusters are employed (THR). The *Target* spacecraft applies a coarse three-axis attitude control based on magnetometers, sun sensors, and GPS receivers (similar to *Main*), with three magnetic torque rods as actuators. The nominal attitude profile for *Target* will be sun or zenith pointing. For completeness the overall GNC sensors and actuators used for attitude and orbit control on *Main* and *Target* are listed in Table 1.

**Tab. 1: Main and Target key sensors and actuators for attitude and orbit control.**

PRISMA	GNC	Main	Target
Attitude	Sensor	MM,SS,GYR, ACC,SCA,GPS	MM,SS, GPS
	Actuator	MT, RW	MT
Orbit	Sensor	GPS,VBS, FFRF,ACC	-
	Actuator	THR	-

### 3.2.2 Space-borne Autonomous Formation Flying Experiment (SAFE)

Within PRISMA, DLR/GSOC has assumed responsibility for providing the GPS-based navigation functionality which comprises the provision of

1. Phoenix GPS receivers
2. Onboard Navigation System for absolute/relative orbit determination
3. On-ground precise orbit determination (POD).

Among the various experiment sets within PRISMA, DLR/GSOC will perform the

1. Spaceborne Autonomous Formation Flying Experiment (SAFE)
2. Onboard Autonomous Orbit Keeping (AOK) of a single spacecraft.

AOK is intended for execution at the end of the PRISMA mission operations phase. In particular, the primary objectives of DLR's contributions to PRISMA are to provide GPS navigation fixes and raw data of *Main* and *Target*, to provide on *Main* a precise absolute orbit solution for *Main*, to provide on *Main* a precise relative orbit solution of *Target* w.r.t. *Main*, to implement a guidance law for a safe separation strategy, to provide a robust control algorithm for formation keeping and reconfiguration, to demonstrate autonomous orbit control of close formations, to implement an automated on-ground process for precise orbit reconstruction. In addition, the secondary objective is to demonstrate an autonomous absolute orbit control of the *MAIN* spacecraft.

The navigation software developed by DLR performs real-time GPS-based absolute and relative navigation in order to support all PRISMA mission phases. Goal of the absolute and relative orbit determination is to achieve an accuracy of 2 m and 0.1 m, respectively (3D, rms) and provide continuous position and

velocity data of the participating spacecraft at a 1 Hz rate for guidance and control purposes as well as for the PRISMA payload. This is achieved by three functional modules residing in the MAIN on-board computer. The three modules are executed at 30 s and 1 s sample times to separate the computational intensive orbit determination task from orbit prediction functions with low computational burden. An extended Kalman filter is applied which processes pseudorange and carrier-phase measurement data issued by the local Phoenix GPS receiver on MAIN and sent via an Inter Satellite Link (ISL) from the remote Phoenix GPS receiver on TARGET.

The control software developed by DLR realizes an autonomous onboard orbit control of the MAIN spacecraft with respect to the TARGET spacecraft. To this end, the concept of relative eccentricity/inclination vector separation of the formation is applied together with an active control of the relative semi-major axis and mean argument of latitude. Current navigation data from the GPS-based navigation modules are used to compute the deviation of the current relative state from the reference relative state in order to generate velocity increments for autonomous maneuver execution. The relative states are based upon a mean orbital elements representation. The control concept applies a simple and robust deterministic maneuver planning to maintain or reconfigure the formation.

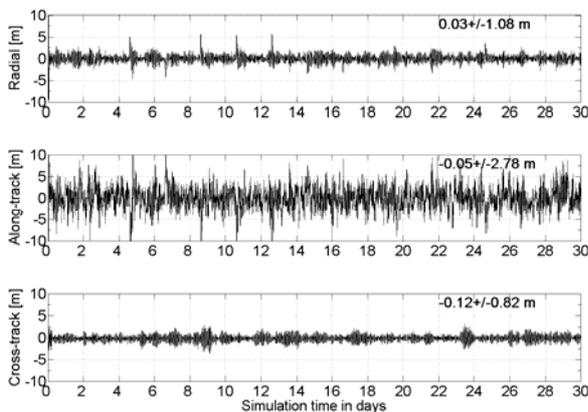
## 4. GNC SYSTEM VALIDATION

The GNC flight software and prototypes are tested and validated at DLR/GSOC as standalone units prior to the full integration into the PRISMA and TanDEM-X spacecraft on-board computers. Thanks to a novel model-based software design, the GNC software can be implemented and executed on different platforms in a fully consistent manner. This allows a seamless transition between off-line, real-time and hardware-in-the-loop tests during the validation phase. In particular off-line simulations are first conducted in a Matlab/Simulink environment on a standard host PC. Here, the prototype flight software is stimulated through different sources of GPS data with an increasing level of realism. Apart from the classical pure software simulations which make use of emulated GPS measurements, usage is made of real GPS

flight data from the Gravity Recovery and Climate Experiment (GRACE) during the closest encounter of the twin satellites or from the TerraSAR-X spacecraft. As a next step real-time hardware-in-loop tests are conducted comprising GPS receivers hardware and a 2x12 channels Spirent GSS7700 GPS Signal Simulator (GSS). Finally the complete application is ported to a Real-Time Executive for Multiprocessor Systems (RTEMS) environment in a FPGA-based LEON3 board, representative of the PRISMA (or TanDEM-X) on-board computer, by means of Matlab/Simulink Real-Time-Workshop. Overall the test and validation process shows the compliance of the GNC software to the challenging requirements of the PRISMA mission in terms of functionality, data interface, GNC accuracy, on-board memory and CPU load and paves the way for the full integrated system level tests with hardware-in-the-loop.

#### 4.1 TanDEM-X (TAFF)

Typical TAFF real-time navigation errors obtained from a long-term real world simulation (30 days) are depicted in Fig. 7. The simulation includes highly realistic sensors (i.e. MosaicGNSS receivers) and actuators (i.e. cold gas thrusters) models. Some spikes are visible in the Fig. 7 especially in the radial component. These effects are caused by the erroneous a-priori information provided to the navigation filter at the instance of large ground-commanded maneuvers (~cm/s). Even though the navigation accuracy is decreased, the filter is shown to be robust enough to absorb these effects.



**Fig. 7: TAFF relative navigation errors in radial (top), along-track (middle) and cross-track (bottom) directions over 30 days of software simulation.**

Overall, the TAFF relative navigation accuracy is at the meter level as required. The performance of the relative orbit control has been assessed by comparing the actual osculating relative motion produced by the reference orbit propagators with the desired osculating relative motion defined by the nominal relative orbital elements. Table 2 summarizes the expected autonomous onboard control performance and compares it with simulation results using a ground-in-the-loop approach (i.e. maneuvers planned from ground every 12-24 hours). Thanks to the shorter reaction time of the controller embedded in the TDX onboard computer the control accuracy is superior, especially in along-track direction.

**Table 1: Achieved control performance over 30 days using autonomous and ground-in-the-loop orbit keeping.**

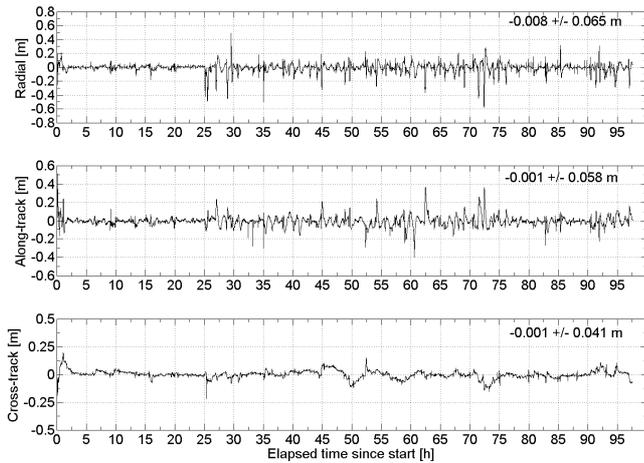
	Control performance, rms [m]		
	Radial	Along-track	Cross-track
TAFF	1.7	6.5	0.3
Ground	2.7	26	0.4

#### 4.2 PRISMA (SAFE)

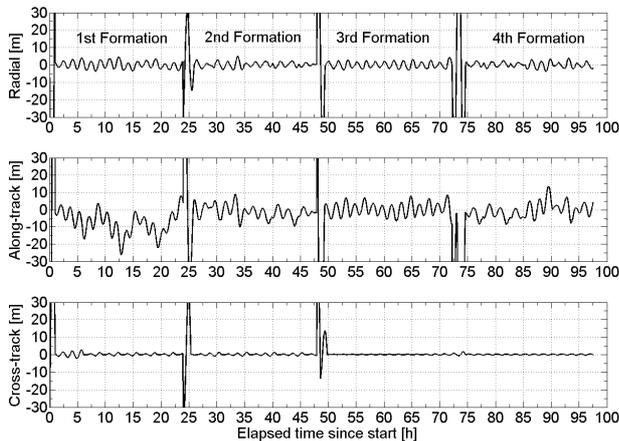
Presently the DLR/GSOC's GNC flight software for PRISMA is being validated through hardware-in-the-loop closed-loop simulations. As an example Fig. 8 and 9 show relevant results obtained from the execution of the flight software in real-time on engineering models of the *Main* and *Target* onboard computer. The integrated system level tests demonstrate relative navigation accuracies at the centimeter level (< 0.1 m, 3D, r.m.s, cf. Fig. 8) and relative orbit control accuracies at the meter level (cf. Fig. 9). The navigation error budget is dominated by the lacking knowledge of the *Target* attitude and thus by the high uncertainty of the GPS antenna offset with respect to the center of gravity. In addition no integer ambiguity resolution and the inclusion of frequent orbit control maneuvers are responsible for the reduced performance relative to Leung et al (2005).

Compared to TanDEM-X, the GPS-based relative navigation accuracy is two orders of magnitude better due to the exploitation of raw pseudorange and carrier phase GPS data. On the other hand the formation keeping performances on TanDEM-X and PRISMA are comparable. This is due to the fact that the same feedback control strategy is applied,

which adopts pairs of impulsive maneuvers executed every 1-2 orbital revolutions.



**Fig. 8: PRISMA relative navigation errors in radial (top), along-track (middle) and cross-track (bottom) directions over 4 days of real-time hardware-in-the-loop validation of flight software.**



**Fig. 9: PRISMA control tracking errors in radial (top), along-track (middle) and cross-track (bottom) directions over 4 days of real-time hardware-in-the-loop validation of flight software.**

In contrast to TAFF, the PRISMA experiment generalizes the control method allowing the maintenance and reconfiguration of arbitrary formation configurations. As shown in Fig. 9, a total of four constellations is acquired and maintained by the control software over the 4 days long real-time test. Large deviations of the control tracking errors are visible at the instance of the formation reconfigurations each 24 hours. The 1<sup>st</sup> constellation in Fig. 9 is controlled through pairs of along-track maneuvers which change the semi-major axis and introduce drifts in along-track direction. The relative orbit control is performed instead through radial

maneuvers in the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> constellation, resulting in an overall improvement of the along-track performance at the expense of higher fuel consumption.

## 5. FUTURE RESEARCH

Future activities at DLR/GSOC in the frame of autonomous formation flying will focus on three main topics:

1. Formation Flying Radio Frequency (FFRF) sensor analysis.
2. Decentralized continuous orbit and attitude control of formation flying spacecraft.
3. Proximity operations, rendezvous and docking of robotic and in-orbit servicing missions.

FFRF offers integrated functionalities like three-dimensional localization, inter-satellite link and multi-satellite synchronization at typical separations between 3 m and 30 km. The adopted GNSS-like metrology provides observables like range and range rate, line-of-sight (LOS) and LOS rate, pseudo code and phase measurements with overall performances at 1 cm level (relative positioning) and 1° (relative orientation) on the line of sight axis. These characteristics make FFRF a perfect candidate for future outer space distributed telescopes. The comparison of GNSS-based relative navigation with results obtained from self-contained formation flying metrology sensors like FFRF is extremely valuable for the validation of these novel technologies. To assist the validation of the FFRF sensor in the PRISMA mission, SSC, CNES and DLR have agreed on a mutual exchange of GPS and FFRF data in that mission.

In order to enable advanced formation flying missions, the tasks of orbit and attitude determination and control should be studied as a single combined problem. Research at DLR/GSOC will focus on the coupling between orbit and attitude dynamics and on combinations of various measurement types issued by navigation devices (e.g. GNSS, FFRF) and attitude sensors (e.g. Star trackers). The problem of decentralized formation control will be studied, using modern control theory to provide optimal orbit/attitude formation control with minimal information passage between the

individual spacecraft and the highest possible level of system redundancy.

Formation flying radio frequency, optical metrology and combined orbit/attitude control represent the key technical challenges to precise and reliable in-orbit servicing or inspection missions. To assist pre-flight simulations of such complex technologies, the build-up of a new robotic simulator for proximity operations is currently prepared by DLR. It will provide a follow-on to the former European Proximity Operations Simulator (EPOS) facility and offer a valuable test-bed for future LEO and GEO servicing missions.

## 5.1 FUTURE MISSIONS

### 5.1.1 PROBA-3

PROBA-3 is the third mission in ESA's Project for Onboard Autonomy. The PROBA spacecraft provide a platform for validating new space systems while carrying an "added value" user payload which can directly benefit from the innovations under test. PROBA-3 will demonstrate the technologies required for formation flying of multiple spacecraft. An instrument to observe the solar corona forms the basis for the ongoing design phase. Formation flying technologies will make new types of missions possible and provide a leap in the performance of future science, Earth observation and application missions. Mastering formation flying missions requires the development of specific technologies well beyond the present state-of-the-art in fields such as metrology and spacecraft Guidance, Navigation, and Control (GNC). PROBA-3, currently in its preparatory study phase, will comprise two independent, three-axis stabilized spacecraft flying close to each other with the ability to accurately control the attitude and separation of the two craft. Utilizing either cold-gas or electrical thrusters for agile maneuvering, and both radio-frequency and optical (laser-based) metrology techniques for accurate position measurement and control, the combined system is expected to achieve a relative positioning accuracy of the order of 1 mm over a separation range of 25 to 250 meters. The PROBA-3 mission represents for DLR/GSOC the natural logical next step after GRACE, PRISMA and TanDEM-X to advance its own capabilities in the formation flying, proximity operations and in-orbiting servicing

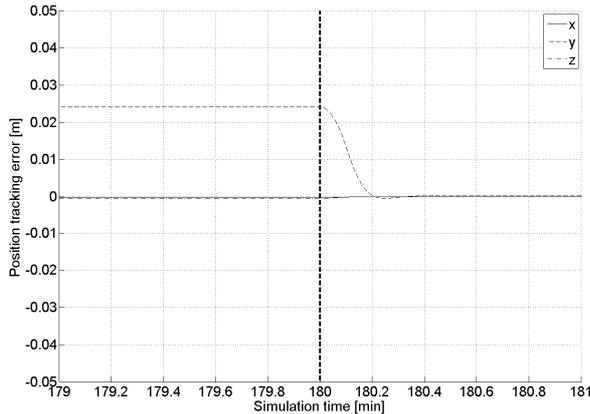
technologies. A technological gap exists between remote sensing LEO formations and outer space distributed satellite systems (e.g. virtual telescopes). The envisaged three-order-of-magnitude improvement of the required metrology and actuation needs requires the implementation of continuous autonomous formation control (in contrast to the sparse maneuvering in LEO) and the development of navigation systems at altitudes above the GNSS constellations.

### 5.1.2 VIRTUAL TELESCOPES

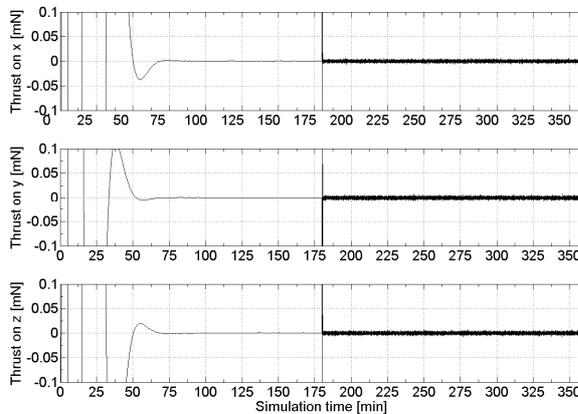
Dual spacecraft telescopes represent a relevant class of future formation flying applications. These instruments aim at the detailed spectral investigation of sources which are too faint for study with the current generation of observatories (e.g. Chandra, XMMNewton). The typical mission profile seeks orbits characterized by a low level of perturbations, stable thermal environment, lack of eclipses, and wide sky visibility. In contrast to the unfavorable LEO environment, in this context optimum conditions are offered by geostationary orbits (GEO), highly elliptical orbits (HEO) and Halo orbits around the Libration points of the Sun-Earth system. Distributed telescopes are composite spacecraft composed of a detector and a mirror unit flying as a formation during science operations. Typical separations aim at focal lengths of the order of 30-100 m. Autonomous formation flying capabilities are driven by the telescope optical design and should allow uninterrupted science observations. This translates into combined attitude/orbit control systems with required navigation accuracies at (sub-)centimeter level.

The X-Ray Observatory (XRO), also known as XEUS (X-Ray Evolving-Universe Spectroscopy), is a relevant example of a dual spacecraft telescope. One of the main science goals of XRO is to investigate the high-redshift Universe. The current mission scenario is based on a Halo orbit around L2 and a composite spacecraft with a focal length of 35 m. The detector satellite is designed to support the payload units and track the focus point of the mirror satellite as to maintain it at the instrument focal plane. The launch of both units as a single stack is planned at the end of 2017, with nominal operations extending until the end of 2022.

Fig. 10 and 11 show preliminary results obtained at DLR/GSOC through the rapid prototyping of an optimum LQR observer-controller at L2 for a formation flying scenario representative of XEUS [Ardaens and D’Amico, 2008]. The system is composed of two satellites flying in close formation around the L2 libration point of the Sun, Earth-Moon system. The leader has a free motion on a halo orbit while the chaser has to keep a user-defined baseline between the two spacecraft. The baseline is set to 100 m for this simulation.



**Fig. 10: Position tracking control error during the transition from coarse to tight relative control during XEUS simulation.**



**Fig. 11: Thrust level required for the tight relative control using an optimum LQR controller.**

The chaser is equipped with three sets of two state-of-the-art electric thrusters aligned with the spacecraft body frame (x,y,z) with a nominal thrust of 30 mN per thruster. The relative position is measured using a fine optical sensor whose expected accuracy is 0.1 mm (longitudinal) and 0.15 mm (lateral). The chosen halo orbit is an unstable halo orbit around L2 (i.e., absolute orbit corrections are

necessary to keep its stability). Since the time scale necessary for the assessment of the controller (some days) is much smaller than the critical time at which the halo orbit loses its stability (several months), no absolute orbit maneuver is performed. A complete revolution around L2 is achieved after about 6 months.

As shown in Fig. 10, established linear control methods can achieve the required sub-centimeter level relative control accuracy. Fig. 11 depicts the required thrust levels for coarse and tight relative control. The necessary thrust resolution is at the  $\mu\text{N}$  level while the thrust saturation limit force is 60mN.

## ACKNOWLEDGMENTS

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